THEA ENERGY

Stability-optimized quasi-axisymmetric stellarator equilibrium

Mike F. Martin[1], D.W. Dudt[1], D.A. Gates[1], W.A. Cooper[2], N.M. Ferraro[3], A.M. Wright[4]

1. Thea Energy 2. EPFL 3. PPPL 4. UW-Madison

Introduction

At the plasma pressures relevant for fusion-grade plasmas, ideal MHD instabilities can quickly destroy plasma confinement on Alfvenic timescales. Stellarator optimization must therefore account for ideal MHD stability in any viable candidate equilibrium. Here, we present quasi-axisymmetric (QA) equilibria optimized for linear, ideal MHD stability. Results are analyzed using the TERPSICHORE [1] code.

Tools

Stellarator equilibria are optimized using the DESC [2] code. The code TERPSICHORE is used within DESC to optimize for MHD stability by minimizing the growth rate

DESC

- Stellarator 3-D MHD equilibrium solver and optimization suite
- Uses pseudo-spectral approach with automatic differentiation
- GPU-accelerated

TERPSICHORE

- Linear, ideal MHD stability eigenvalue solver in 3-D geometries
- Uses the Energy Principle to determine stability
- Vacuum region with conformal-like conducting wall
- User-specified active mode table
- Internal/external pressure- and current-driven instabilities

$$\delta W_p + \delta W_v - \omega^2 \delta W_k = 0$$

Plasma Potential Energy — Vacuum Region Energy — Plasma Kinetic Energy

- Plasma is **stable** when $\omega^2 > 0$
- Displacement (perturbation) vector is Fourier decomposed in the radial and binormal directions

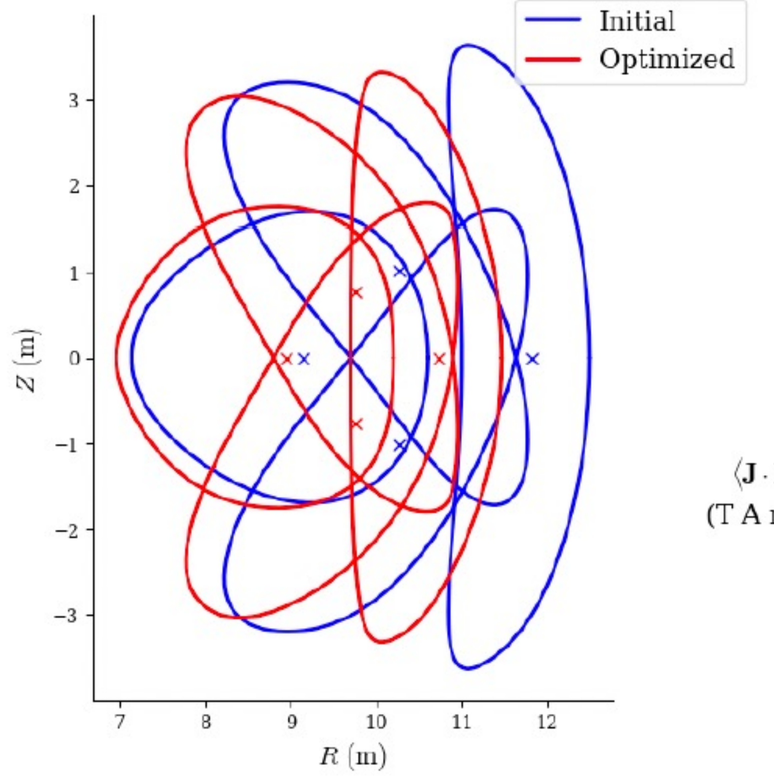
Radial displacement vector

$$\xi(s,\theta,\phi) = \sum_l \xi_l(s) \sin(m_l\theta - n_l\theta)$$

Selected Equilibria

Using the Landreman, Buller, Drevlak [3] finite beta, two-field period QA configuration as an initial condition, the equilibrium is optimized for MHD stability using TERPSICHORE

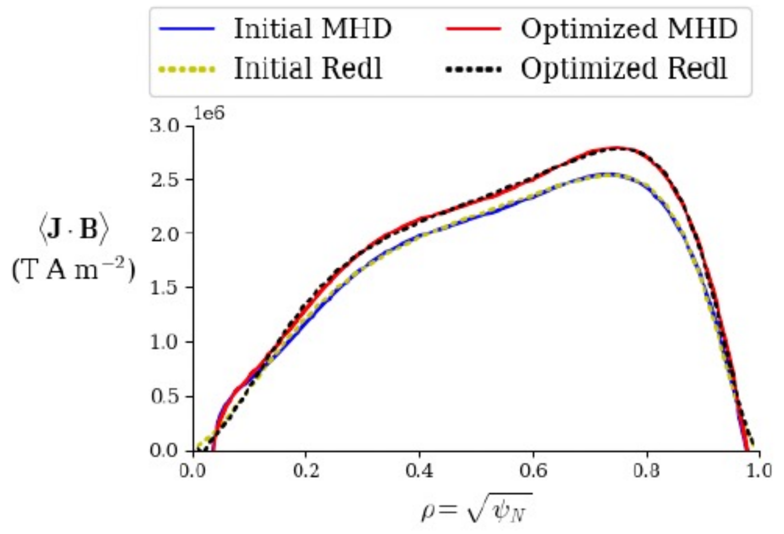
See poster JP12.00055 in this session for details



Aspect Ratio: $\frac{R_0}{a}$

Initial $= 6.00$

Optimized $= 5.61$



TERPSICHORE Growth Rate (normalized to Alfven frequency)

$\gamma^{initial} = 8.1 \cdot 10^{-3}$

$\gamma^{optimized} = 5.5 \cdot 10^{-3}$

QA Error: $\frac{\sqrt{\sum_{n\neq0} B_{mn}^2}}{\sqrt{\sum_{m,n} B_{mn}^2}}$

Initial $= 4.4 \cdot 10^{-3}$

Optimized $= 2.6 \cdot 10^{-3}$

TERPSICHORE Setup

Simulation parameters:

- Conformal wall with a minor radius of 2x plasma minor radius
  - "No wall" limit
- Boozer spectral range: $m_{boz} = 17,\ |n|_{boz} = 16$
- 256 radial surfaces

Active Modes

- Restricted to $m_{stab} \leq 12,\ |n|_{stab} \leq 4$
- Simulations are run using either all modes, N=0 family, or N=1 family

Stability Criteria

- Positive growth rates are not necessarily catastrophic [4]
  - Growth rates above ~1% of an Alfven time are indeed considered serious
  - Behavior slightly below this range is more ambiguous and requires nonlinear verification

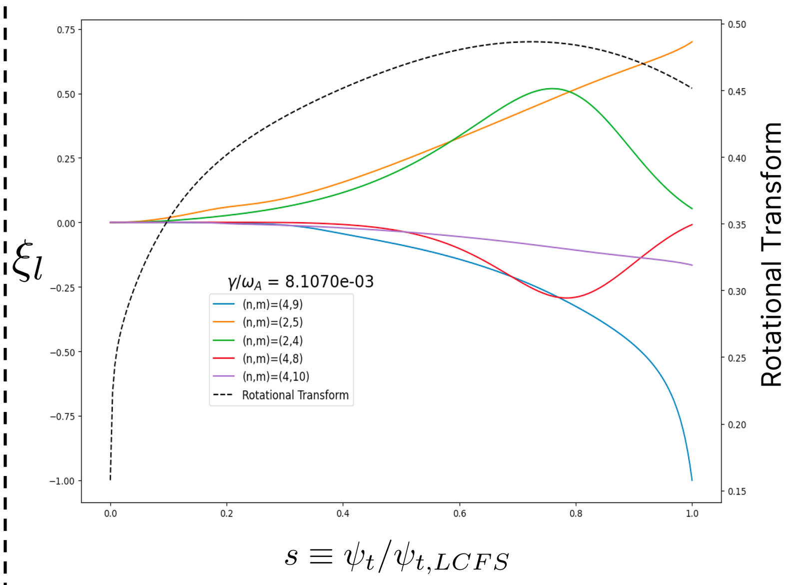
Radial Displacement Eigenfunction

- The eigenfunctions of the radial displacement vector amplitude of the **fastest growing mode** is plotted as a function of normalized toroidal flux
- Plotted magnitudes are normalized to the maximum value of all eigenfunctions
- The N=0 family (periodicity-preserving) and N=1 family (periodicity-breaking) are analyzed separately for each equilibrium

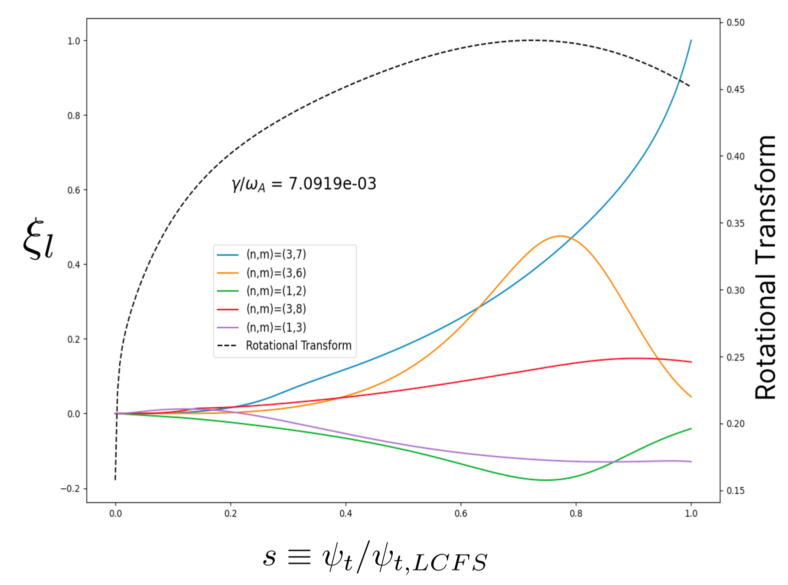
(only showing 5 largest eigenfunctions)

Initial Equilibrium

N=0 Family

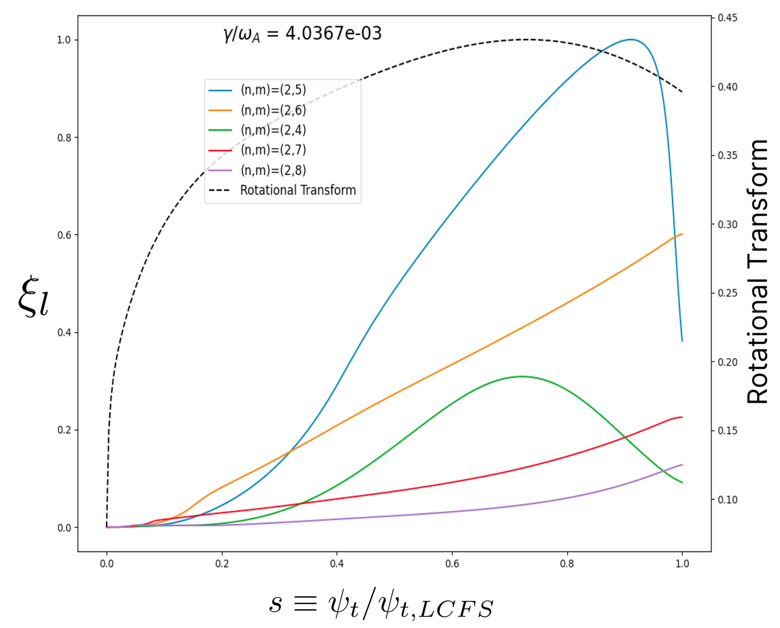


N=1 Family

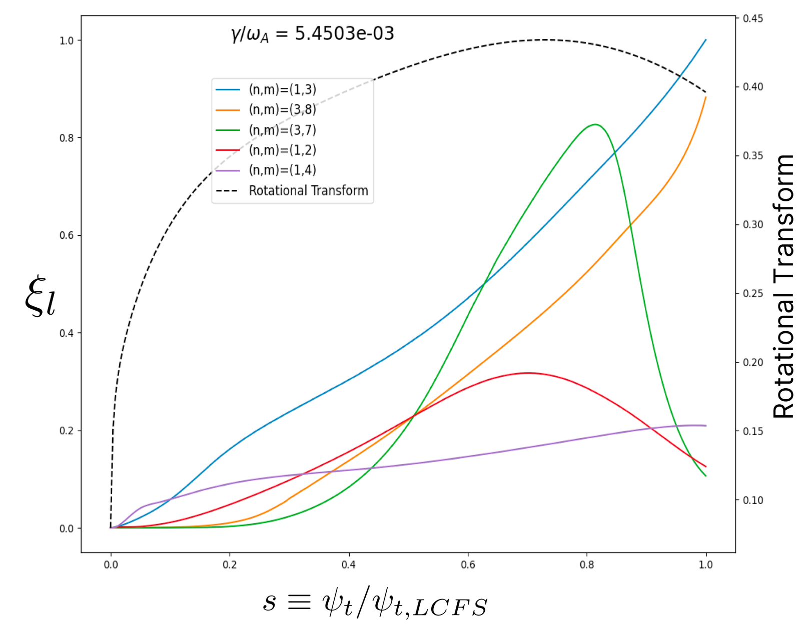


Optimized Equilibrium

N=0 Family



N=1 Family

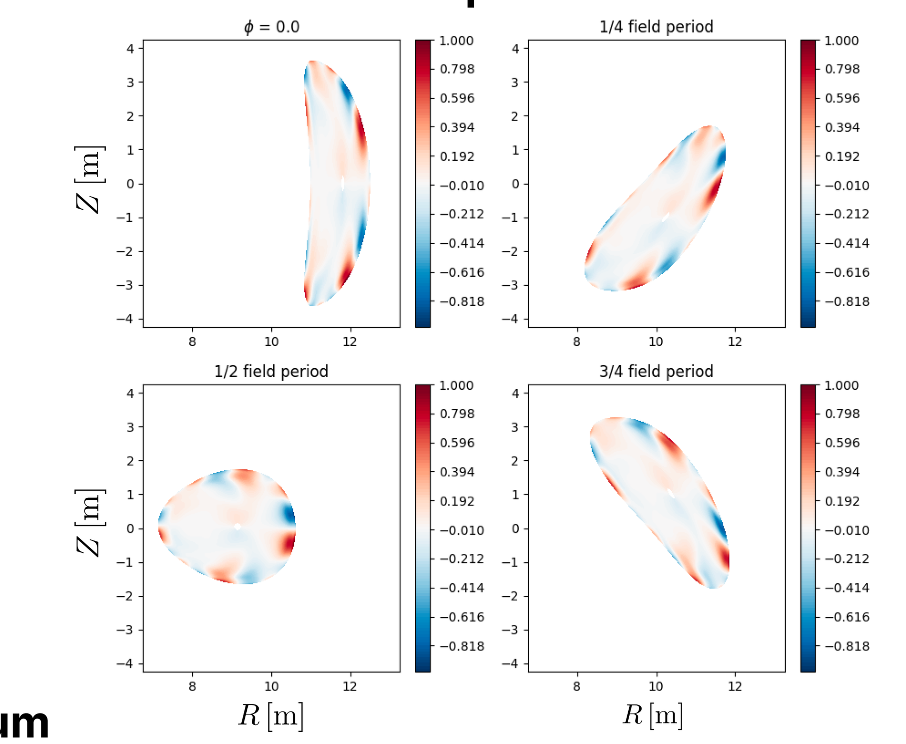


- Dominant eigenfunctions in both equilibria are global and external
- The dominant mode structure and family changes after optimization
- Growth rates are very close to the threshold where results become ambiguous

Radial Perturbation Contours

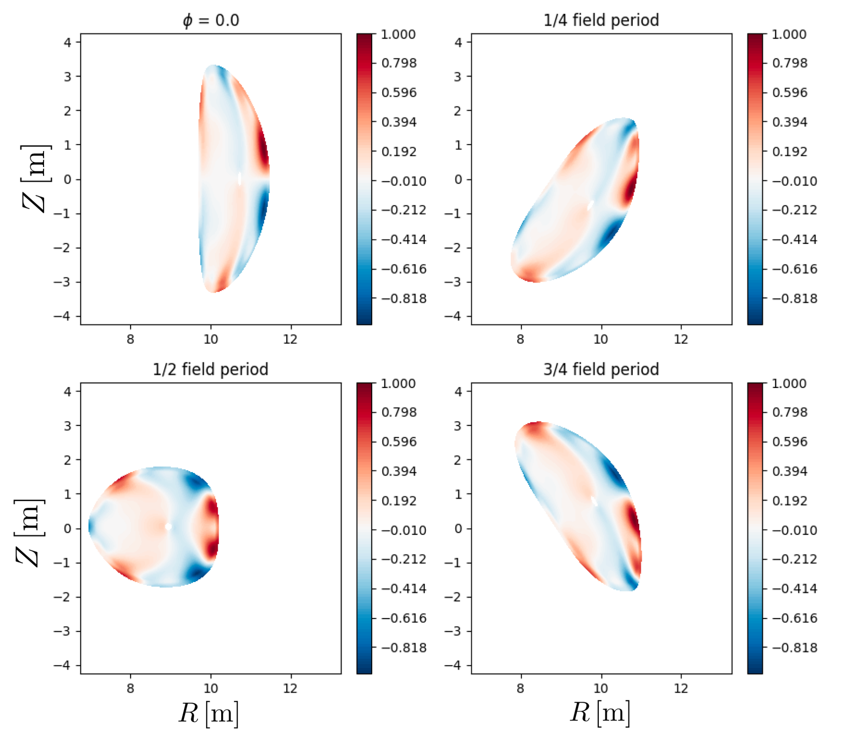
- TERPSICHORE provides the displacement vector of the fastest growing mode
- $\xi/\max(|\xi|)$ is plotted on each surface
  - inward/outward

Initial Equilibrium



- Initial equilibrium has a dominant (n,m) = (-4,9) instability
- Strongly edge localized and ballooning

Optimized Equilibrium



- Optimized equilibrium is dominated by a (n,m)=(-1,3) instability
- Also ballooning, but more radially extended than the original equilibrium

Conclusion / Future Work

- The growth rate for MHD instabilities has been successfully reduced using TERPSICHORE in a DESC optimization
- Work is ongoing to further reduce the growth rate and perform subsequent nonlinear simulations
- Stay tuned!

[1] D.V. Anderson et al., 1990 Int. J. Supercomput. Appl. 4 34
[2] D.W. Dudt et al., 2023 J. Plasma Phys. 89 955890201
[3] M. Landreman et al., 2022 Phys. Plasmas 29 082501
[4] A.D. Turnbull et al 2011 Nucl. Fusion 51 123011

www.thea.energy
info@thea.energy