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Single-Stage Optimization for Reactor-Relevant Designs

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Motivation

A stellarator fusion reactor must be optimized for a variety of physics and engineering objectives to be commercially viable, including but not limited to:

- good particle confinement
- stable dynamics
- low turbulent transport
- feasible coil geometry

The traditional "two-stage" approach of optimizing a fixed-boundary equilibrium first and then optimizing the external coils afterwards is time-intensive and can lead to sub-optimal results [1, 2].

Simultaneously optimizing both the plasma boundary and the coil parameters in a "single-stage" approach has been shown to better satisfy all the design criteria for the resulting free-boundary solution [3].

We present a new single-stage optimization algorithm that has been developed in the DESC optimization suite [4] to accommodate multi-objective optimization problems with realistic considerations for a commercial power plant.

Stability Optimization with TERPSICHORE

TERPSICHORE is an ideal linear MHD stability solver for 3D configurations with nested flux surfaces, and the vectorized Fortran code runs sufficiently fast to be used within an optimization loop [5].

DESC can call TERPSICHORE to minimize the growth rate γ of the most unstable mode, thus stabilizing an equilibrium ($\gamma < 0$ indicates stability).

Each call from DESC involves writing a TERPSICHORE input file and a VMEC-style "wout" equilibrium file, but only the required outputs are written by DESC to reduce computational costs.

The gradient of the growth rate with respect to the optimization variables is approximated from forward finite differences with a fixed step size and combined with the other Jacobians in DESC computed by automatic differentiation.

The number of TERPSICHORE calls required for the finite differencing scales linearly with the number of optimization variables but can be run in parallel.

References

[1] D. Williamson et al., 2005 *Fusion Eng. Des.* **75** 71

[2] M. Drevlak, 1998 *Fusion Tech.* **33** 106

[3] R. Jorge et al., 2023 *Plasma Phys. Control. Fusion* **65** 074003

[4] D.W. Dudt et al., 2023 *J. Plasma Phys.* **89** 955890201

[5] D.V. Anderson et al., 1990 *Int. J. Supercomput. Appl.* **4** 34

[6] M. Landreman et al., 2022 *Phys. Plasmas* **29** 082501

[7] F. Najmabadi et al., 2008 *Fusion Sci. Tech.* **54** 655

Single-Stage Coil Optimization

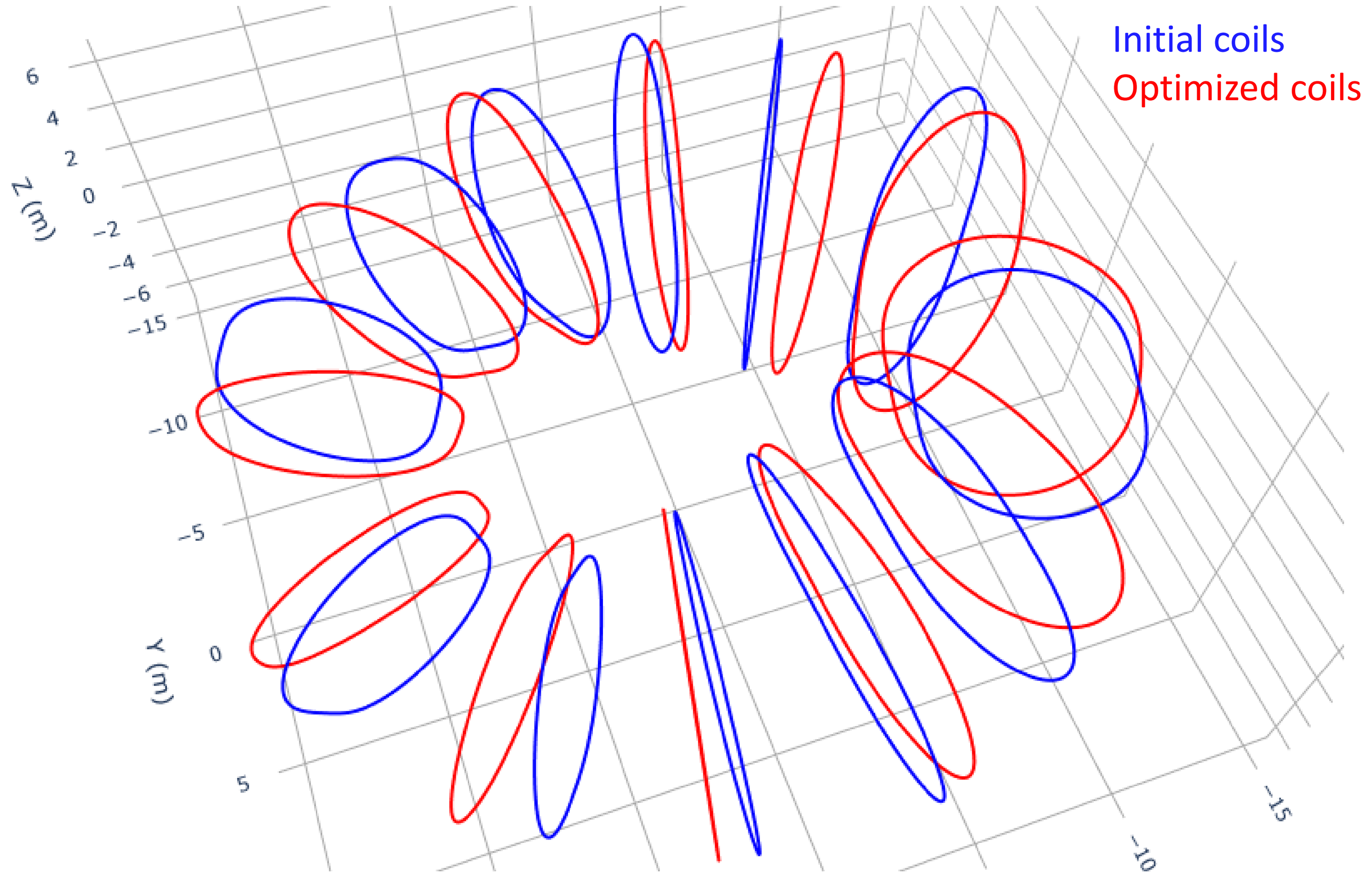
Several different coil parameterization classes are available in DESC, including planar coils with the following parameterization:

- Centroid position — $\boldsymbol{x}_0$ — (Cartesian or cylindrical coordinates)
- Plane normal — $\boldsymbol{n}$ — (Cartesian or cylindrical coordinates)
- Radius — $r(\theta)$ — (represented as a Fourier series)

The following coil objectives have been implemented in DESC:

- Length — L — (also Current-Length)
- Curvature — κ — (signed to also indicate convexity)
- Torsion — τ — (identically 0 for planar coils)
- Coil-Coil Distance — Δ_{coil}
- Plasma-Coil Distance — Δ_{plasma}
- Quadratic Flux — $\boldsymbol{B} \cdot \boldsymbol{n} = 0$
- Pressure Balance — $B^2_{\rho=1^+} - B^2_{\rho=1^-} = p$
- Sheet Current — $\mu_0 \nabla\Phi = \boldsymbol{n} \times \left(B_{\rho=1^+} - B_{\rho=1^-}\right)$
- Linking Current — $\sum I_c = \frac{2\pi}{\mu_0} G$

All these objectives use vectorization and automatic differentiation to efficiently compute their values and derivatives.



Example – Initial Equilibrium

The quasi-axisymmetric (QA) stellarator with self-consistent bootstrap current from Landreman, Buller & Drevlak [6] was optimized for planar encircling coils and improved stability in a single-stage optimization.

This equilibrium is at the scale of the ARIES-CS reactor [7]:

- Field Periods $N_{FP} = 2$
- Pressure $\langle\beta\rangle = 2.5\%$
- Aspect Ratio $\frac{R_0}{a} = 6$
- Density $n(\rho) = n_0\left(1 - \rho^{10}\right)$
- Minor Radius $a = 1.7$ m
- Temperature $T(\rho) = T_0\left(1 - \rho^2\right)$
- Field Strength $\langle B\rangle = 5.86$ T

Three (3) unique encircling coils were optimized assuming stellarator and field period symmetry, with the following targets:

- $L \leq 32$ m
- $\kappa > 0$ m^{-1}
- $\Delta_{\text{coil}} = 2$ m
- $\Delta_{\text{plasma}} = 2$ m

Example – Optimization

In addition to the coil objectives, the optimization included objectives for:

- Target a, $\langle B\rangle$, and $\langle\beta\rangle$ at their initial values
- Quasi-Axisymmetry using the two-term metric f_C [4]
- Bootstrap current self-consistency using the Redl formula [6]
- Ideal MHD linear stability using TERPSICHORE [5]

After the single-stage optimization, the bootstrap current was re-optimized for self-consistency and then the coils were also re-optimized.

The optimization ran in about 3.5 hours on a NVIDIA A100 GPU.

The stability calculations included the modes $M \leq 12$, $N \leq 4$.

Conclusions & Future Work

TERPSICHORE has been coupled to DESC to enable ideal linear MHD stability optimization for stellarators.

Single-stage optimization in DESC is computationally efficient and can yield better solutions for both plasma and coil objectives:

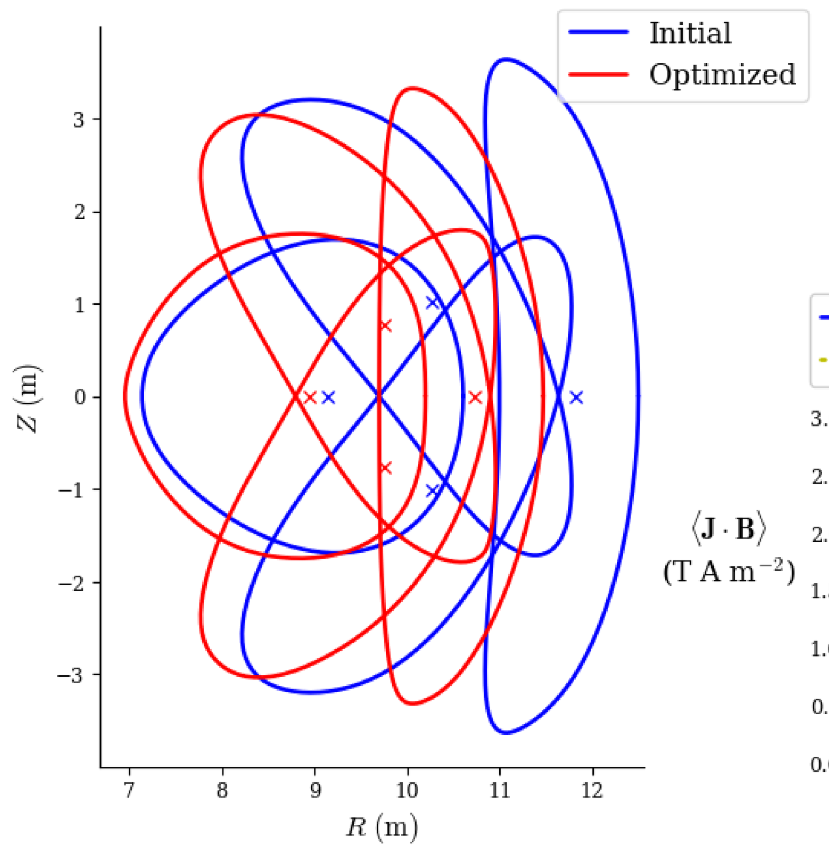
- Improved quasi-symmetry
- Improved stability
- Lower normal field errors from coils

Work is ongoing to incorporate higher-fidelity objectives, including:

- Finite build coils
- Neoclassical confinement
- Turbulent transport

Example – Results

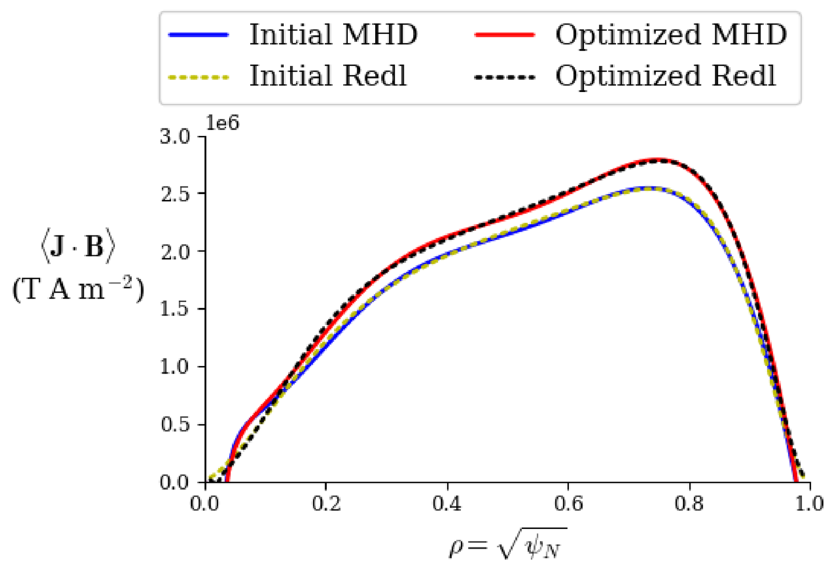
Stability improvement: $\gamma_{\text{initial}} = 7.4 \times 10^{-3} \longrightarrow \gamma_{\text{optimized}} = 5.3 \times 10^{-3}$

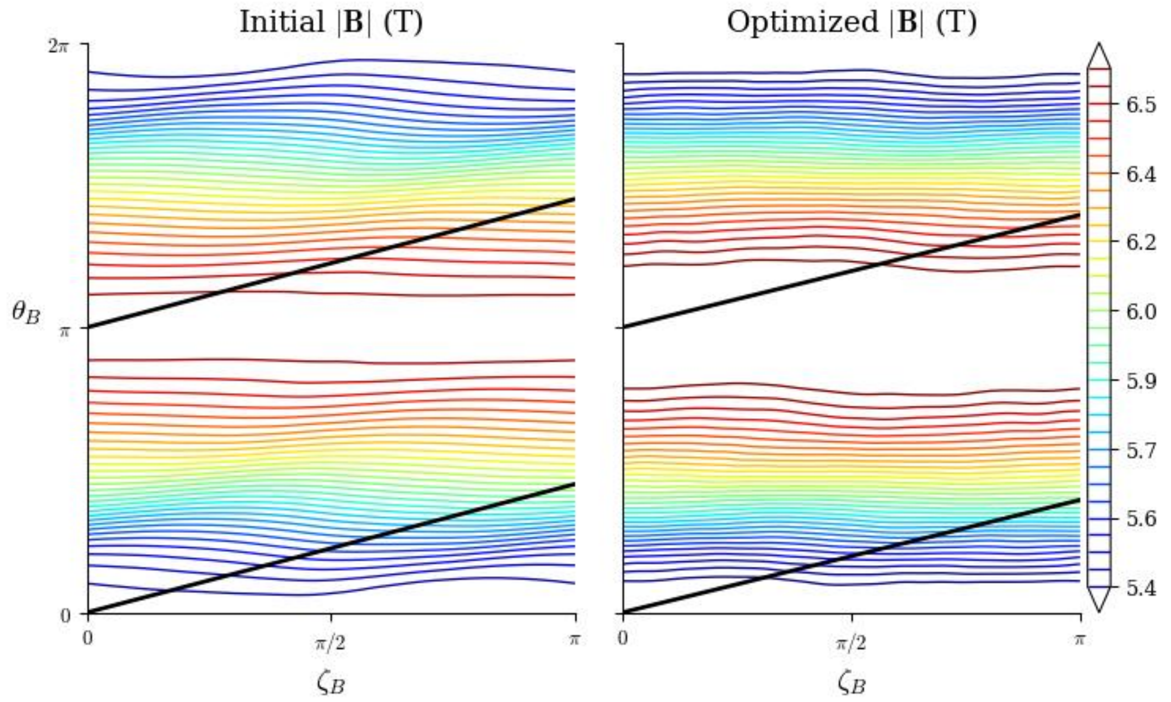


Aspect Ratio: $\frac{R_0}{a}$

Initial $= 6.00$

Optimized $= 5.61$



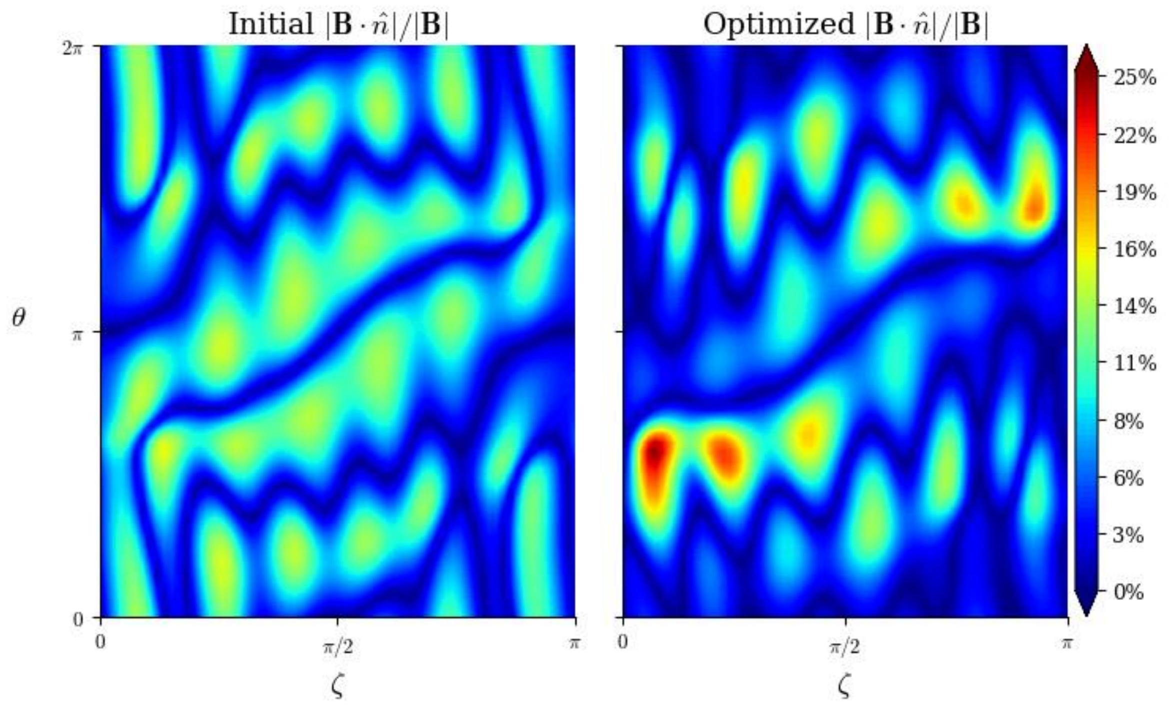


QA error:

$$\frac{\sqrt{\sum_{n\neq 0} B_{mn}^2}}{\sqrt{\sum_{m,n} B_{mn}^2}}$$

Initial $=$ 4.4×10^{-3}

Optimized $=$ 2.6×10^{-3}



Average field error:

$$\left\langle \frac{|\boldsymbol{B} \cdot \hat{\boldsymbol{n}}|}{B} \right\rangle$$

Initial $=$ 6.36%

Optimized $=$ 5.04%

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