THEA ENERGY

Limitations of SPEC convergence in strongly-shaped plasmas

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Introduction

- SPEC[1] (Stepped Pressure Equilibrium Code) is an MHD equilibrium solver that is capable of modeling flux surfaces, magnetic islands, and chaos using MRxMHD[2]
- SPEC is used to characterize the magnetic field topology of a particular quasi-axisymmetric (QA) ideal MHD equilibrium
- Using an incremental approach, we show that the number of volumes, N_ν can be made arbitrarily large, approaching the ideal limit
- If the pressure is slowly incremented from a large N_ν vacuum solution, a finite beta solution can be obtained
- However, we find that convergence only extends to lower resolution plasma boundaries, at least for this equilibrium

Background

SPEC Overview

- The full plasma volume is subdivided into N_ν sub-volumes separated by 'ideal' interfaces
- Within each volume, the thermal pressure is constant and $\mathbf{B}_\nu \cdot \hat{\mathbf{n}} = 0$ at the interfaces, but the total pressure is constant across interfaces

$$[[p + B^2/2]] = 0$$

- The magnetic field in each volume is assumed to undergo Taylor relaxation[3], obeying the Beltrami equation

$$\nabla \times \mathbf{B}_\nu = \mu_\nu \mathbf{B}_\nu$$

- SPEC attempts to extremize a modified version of the MHD energy functional, where the magnetic helicity in each volume is conserved

$$\mathcal{F} = \sum_{\nu=1}^{N_V} [W_\nu - \mu_\nu (K_\nu - K_{o,\nu})/2] \qquad W \equiv \int_V \left(\frac{p}{\gamma - 1} + \frac{B^2}{2}\right) dv \qquad K \equiv \int_V \mathbf{A} \cdot \mathbf{B}\, dv$$

- Variations in μ_ν, $\Delta\psi_{\nu,p}$, the interface harmonics, and magnetic vector potential harmonics represent the possible degrees of freedom available to extremize

$$R_\nu(\theta,\phi), Z_\nu(\theta,\phi) \qquad A_\theta(\theta,\phi) = \sum_i \sum_{l=0}^{L} A_{\theta,i,l} T_l(s) \cos(m\theta - nN_{FP}\phi)$$

$$X(\theta,\phi) = \sum_{m=0}^{M} \sum_{n=-N}^{N} X_{mn,c} \cos(m\theta - nN_{FP}\phi) + X_{mn,s} \sin(m\theta - nN_{FP}\phi)$$

  - SPEC uses Legendre polynomials for the vector potential, except for the innermost volume, which uses Zernike polynomials

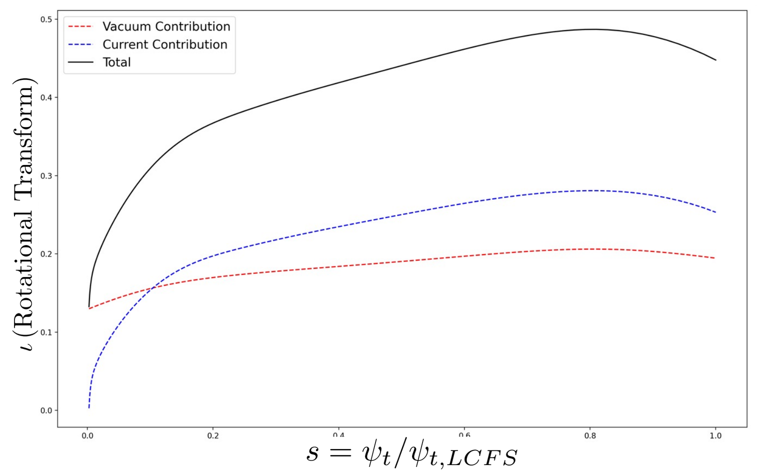
Methods

Step 1 – Find vacuum/current contributions to rotational transform

- Starting with a converged, finite-β DESC or VMEC solution, split the rotational transform profile into its vacuum and current contributions[4].

$$\iota = \frac{1}{S_{11}} \left(\frac{\mu_0 I}{\psi_t'} - S_{12} \right)$$

S_{11}, S_{12} - signed geometric factors



Step 2 – Low volume, low resolution vacuum solution

- Considering the *finite pressure* rotational transform, select a small number of interfaces to lie on flux surfaces that are sufficiently far from low-order rational surfaces.
- Constrain the rotational transform at the interfaces in SPEC
- Choose the noble irrational closest to the vacuum rotational transform at a given toroidal flux

$$\iota_{noble} = \frac{p_1 + \gamma p_2}{q_1 + \gamma q_2} \qquad \gamma = \frac{1+\sqrt{5}}{2}$$

Step 3 – Incrementally increase the number of interfaces

- Using the interface from the converged solution as an initial guess, add an additional interface at $\psi + \Delta\psi_t$, where $|\Delta\psi_t| < 0.01(\psi_t/\psi_{t,LCFS})$
- Repeat until regions outside of low-order rationals are sufficiently dense with interfaces, approximating a continuous pressure profile

Step 4 – Incrementally increase pressure

- The pressure in each volume is approximated as the average of the VMEC/DESC pressure profile over the range of toroidal flux in that volume
- Using a scale factor, slowly bring up the pressure and the current contribution to ι, so that SPEC is constrained to the intermediate rotational transform

$$\iota_{int} = \iota_{vacuum} + \varepsilon_p \iota_{current} \qquad \varepsilon_p \in [0,1]$$

Step 5 – Incrementally increase resolution

- Introduce an extra computational boundary mode $m_{cb}, |n|_{cb} \le X + 1$, and slowly scale up the amplitude of the new R_{mn}, Z_{mn}
- Repeat until resolution of initial VMEC/DESC equilibrium is reached

Results

- We use a scaled version of the 2 field period quasi-axisymmetric (QA) from[5] utilizing the same pressure profile

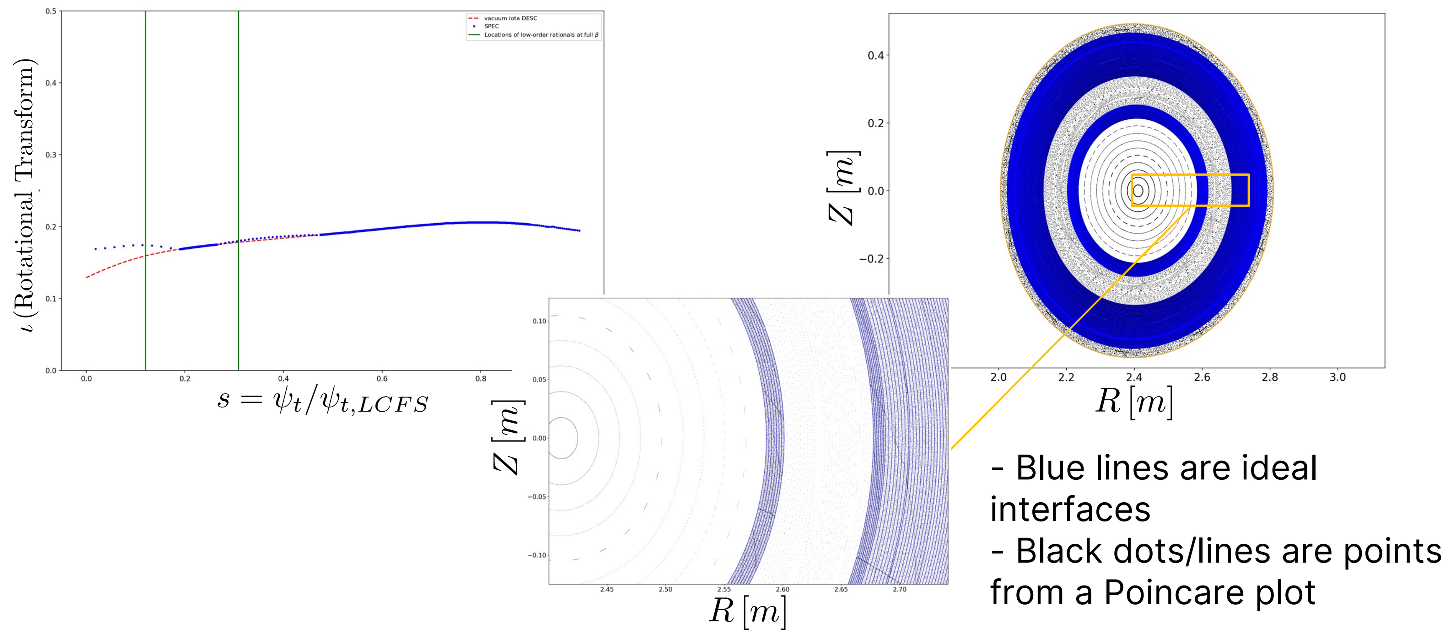
$$n_\alpha(s) = n_{\alpha,0}(1 - s^5) \qquad T_\alpha(s) = T_{\alpha,0}(1 - s)$$

$$R/a = 6 \qquad B_0 = 6\,\mathrm{T} \qquad R = 2.7\,\mathrm{m}$$

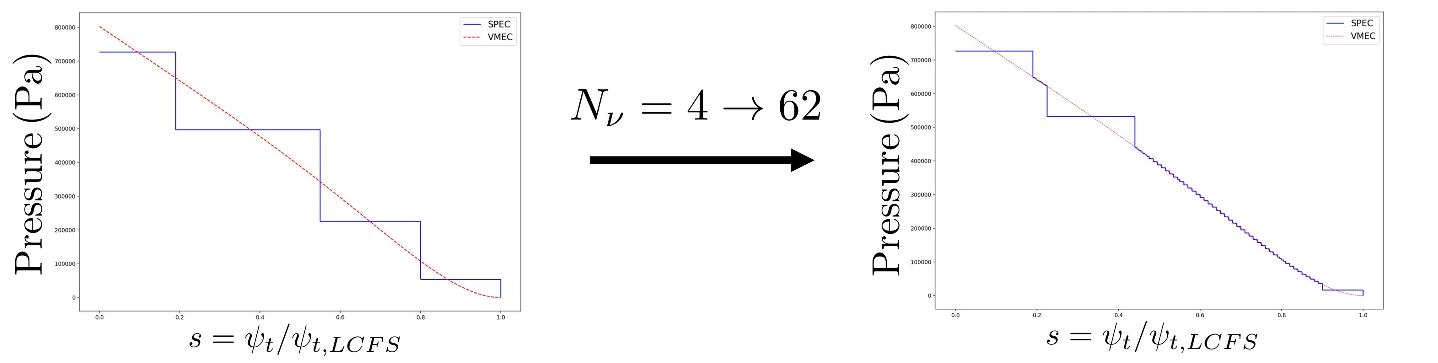
- The SPEC computational boundary (CB) is initialized using only the $m_{cb}, |n|_{cb} \le 2$ modes of the $R_{m,n}$ and $Z_{m,n}$ describing the equilibrium boundary
- SPEC resolution for the following results satisfy the conditions

$$m_{spec} \ge m_{cb} + 6 \qquad n_{spec} \ge |n_{cb}| + 4 \qquad L_{spec} \ge m_{spec} + 5$$

Vacuum – N_ν=62

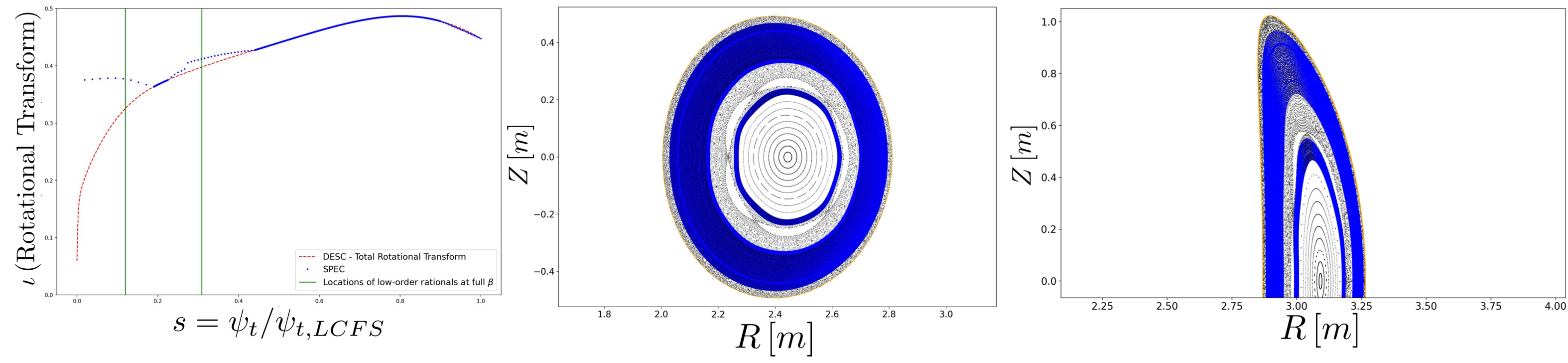


- A well-converged vacuum solution is found where the magnetic field is left unconstrained in regions where islands are expected to develop, while interfaces are densely packed elsewhere
- The SPEC pressure profile becomes effectively continuous in the dense interface region



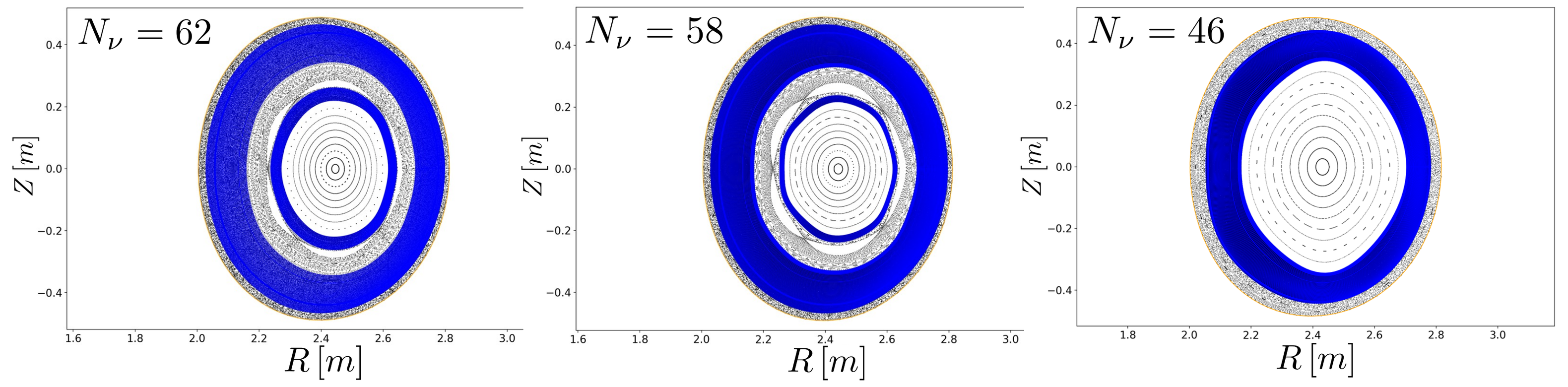
$\beta = 2.5\%$ – N_ν=62

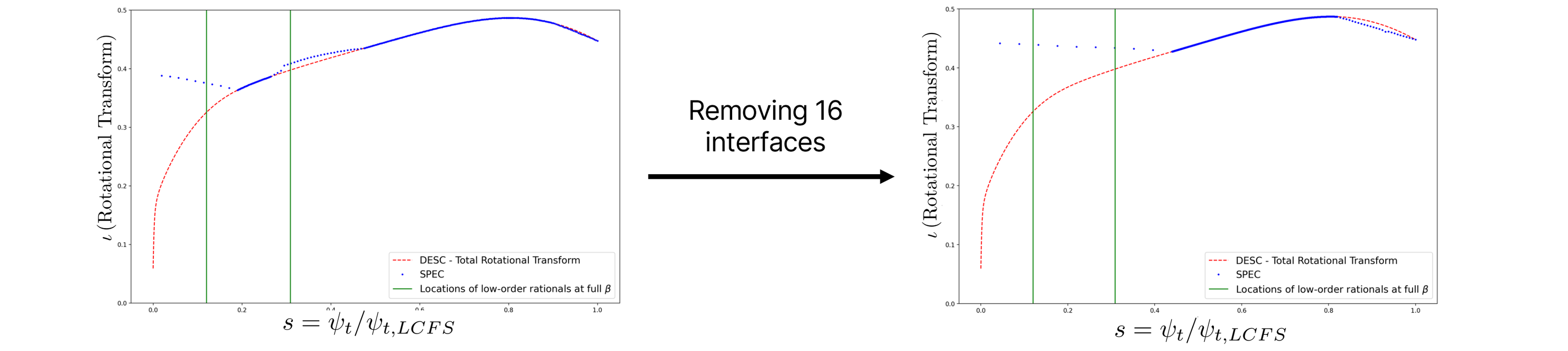
- By incrementally increasing the pressure, a well-converged solution is obtained for the DESC equilibrium beta of $\beta = 2.5\%$
- There is both a clear Shafranov shift, and the appearance of a 2/5 island chain near $\iota = 0.4$



Full Beta – Scaling CB Modes / Modifying Interfaces

- We next expand the set of CB modes to $m_{cb}, |n|_{cb} \le 3$, and incrementally scale up the amplitudes of the new harmonics
- It is found that the solution cannot convergence beyond ~30% of its nominal amplitude
- Modifications to the number/location of surfaces can make minor improvements to the convergence limit, however this leads to increasingly dissimilar rotational transform profiles compared DESC





Conclusions / Future Work

- SPEC solutions for a non-axisymmetric geometry were obtained at finite pressure with large N_ν
- Convergence is extremely sensitive to the CB resolution
- Free boundary SPEC simulations could help eliminate some artificial constraining of the plasma boundary
- Considering the sensitivity of islands to slight changes in the plasma boundary, characterizing the topology for realistic plasma shapes and profiles is not possible with the current state of SPEC

References

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